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OPTIMAL MATERIAL PARAMETERS CHOICE FOR COMPOSITE SHAFTS MANUFACTURING

Abstract: *In past decades, various multi-criteria decision-making (MCDM) techniques have become effective tools for determining the optimal input parameters of various processes. An overview of the application of various MCDM techniques in solving the problem of parametric optimization of the processing process gives a concise picture and the possibility of easier selection of a certain method in the decision-making process. This paper shows the choice of optimal parameters for composite shaft manufacturing. The composite shaft has with ring cross-section, and it consists of aluminum as the base material with a coating of different fiber types covered with epoxy. For optimal parameter choice, different MCDM techniques are used. The most influential parameters are determined. The paper concludes with discussion of the used parameters with guidelines for optimal parameter choice for composite shaft manufacturing.*

Keywords: *Composite shaft, Optimization, MCDM techniques*

1. Introduction

Composite materials nowadays play a vital role in engineering applications such as automotive, aerospace, and other industrial areas. In the last 30 years, various multi-criteria decision-making techniques (MCDM) have become efficient tools for determining optimal input parameters for different processes. A review of the application of various MCDM techniques in solving the problem of parameter optimization in processing provides a concise overview and facilitates the easier selection of a specific method in the decision-making process (Chakraborty & Chakraborty, 2022). Material selection is one of the crucial activities in the product development and design process. MCDM involves defining and evaluating alternatives, establishing criteria, assessing criterion weighting coefficients, and applying ranking systems. Each criterion is associated with the goal stated in the decision-making context, and normalization is used to convert different criteria into compatible measurements (Deng & Edwards, 2007; Edwards & Deng, 2007; Rao, 2008). Inappropriate material selection can lead to damage or failure of the assembly and a significant reduction in performance (Jahan, Mustapha, Ismail, Sapuan, & Bahraminasab, 2011).

The most applied technique for material selection in a specific application area is a hybrid method that combines two or more MCDM methods, as concluded in the paper of (Emovon & Oghenenyerovwho, 2020). Furthermore, it is stated that the most commonly applied decision criterion for selecting the optimal alternative is cost. The highest number of articles on material

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selection was published in 2013, with the most significant journal being Materials and Design. Finally, the country with the highest application of MCDM methods is India (Emovon & Oghenenyerovwho, 2020).

Using AHP and TOPSIS multi-criteria decision-making methods, the process of selecting the appropriate material for the development of horizontal wind turbine blades has been carried out (Okokpujie et al., 2020). Through the integration of three different multi-criteria decision-making methods—Fuzzy Best-Worst Method (finding subjective weights), Criteria Importance Through Inter-criteria Correlation (finding objective weights), and Mixed Aggregation by Comprehensive Normalization Technique—ranking of insulation materials has been conducted to improve energy efficiency in buildings (Aksakal, Ulutaş, Balo, & Karabasevic, 2022).

Integrated multi-criteria decision-making (MCDM) and life cycle assessment (LCA) methods are used for selecting the most suitable material for hydrocarbon pipeline projects (Karamoozian et al., 2023). (Nicolalde, Martínez-Gómez, & Vallejo, 2022) has developed an alternative to material selection by using multi-criteria decision-making methods as tools for optimizing part selection in the automotive industry. The analysis considers not only the technical characteristics of the materials but also environmental aspects, with both being analyzed from an objective perspective, making it distinct from traditional selection methods.

The cathode represents the most significant part of lithium-ion batteries. The assessment of cathode materials is explored based on a set of economic, environmental, and tactical criteria in three main phases. In the first phase, hybrid multi-criteria decision-making (MCDM) methods based on subjective, objective, and combined weights, including SAV-AHP, SAV-CILOS, SAV-AHP-CILOS, TOPSIS-AHP, TOPSIS-CILOS, TOPSIS-AHP-CILOS, CoCoSo-AHP, CoCoSo-CILOS, CoCoSo-AHP-CILOS, MARCOS-AHP, MARCOS-CILOS, and MARCOS-AHP-CILOS methods, are developed, and the performance of cathode materials is evaluated based on the final ranking of alternatives (FRA) and Copeland methodology. In the second phase, using efficient and inefficient boundaries determined by the Data Envelopment Analysis (DEA) model, the performance of cathode materials is assessed. In the third phase, the efficiency of the three proposed methodologies is compared. The results of this study indicate that the hybrid MCDM-FRA methodology is more flexible than the modified DEA model and the hybrid MCDM-Copeland method for solving the problem of selecting a sustainable cathode material (Tajik, Makui, & Tosarkani, 2023).

In the study on ranking aluminum-coconut shell composites, integrated Multi-Criteria Decision-Making (MCDM) approaches such as AHP-TOPSIS and AHP–MOORA are employed. The weight for each criterion is calculated using the Analytic Hierarchy Process (AHP) method and is utilized in the TOPSIS and MOORA approaches for material ranking (Raju, Murali, & Patnaik, 2020).

A new hybrid method, AHP/CRITIC-COPRAS, has been applied for selecting the optimal alternative material for automotive components. The weight of each assessed material is determined by establishing the criteria's importance using the Criteria Importance Through Inter-criteria Correlation (CRITIC) method and the Analytic Hierarchy Process (AHP). Alternative ranking is evaluated using the Complex Proportional Assessment (COPRAS) method (Aherwar, Pruncu, & Mia, 2022).

During the material selection process, designers must possess a thorough knowledge of the properties of the considered materials and their behavior under working conditions, such as strength, durability, flexibility, weight, ability to cast, machinability, electrical conductivity, etc. Using the VIKOR method, ranking and selection of the best parameter for motorcycle shaft production have been performed (Virdi & Saini, 2014). The new model of the VIKOR method is capable of reducing the risk involved in selecting appropriate materials according to a set of predefined criteria, as explained by (Jahan et al., 2011). The results of applied examples have demonstrated the potential of the proposed VIKOR in Multi-Criteria Decision Making (MCDM), aiding designers and decision-makers in making stronger decisions, especially in applications for selecting biomedical materials (Sharma et al., 2015).

Before the production of any product, dominant parameter choices are considered to minimize the chances of system failure due to unplanned arrangements. The material selection problem can be treated as a Multi-Criteria Decision Making (MCDM) problem. (Sharma et al., 2015) explore a ranking method for prioritizing parameters for motorcycle shaft production from various criteria using AHP and ANP (Sharma et al., 2015). The use of SAW, TOPSIS, and MOORA methods is quite suitable and computationally easy for evaluating and selecting the appropriate material from a given set of alternatives. These methods use measures of the considered criteria with their relative importance to arrive at the final ranking of alternative gear materials. Therefore, these popular MCDM methods can be successfully applied to solve any decision-making problem with any number of criteria and alternatives in the manufacturing domain (Prithwiraj Jana, 2000).

(Rahim, Musa, Ramesh, & Lim, 2020) systematically explore available methods, tools, and proposed strategies for material selection in their review. Two important themes have emerged, screening and "selection and ranking" of materials in the selection process. AHP, TOPSIS, and VIKOR are methods that have stood out in recent years in the process of ranking and selecting materials.

This paper presents the optimal selection of parameters related to the type of composite material properties for a truck composite drive shaft. The mentioned shaft was originally made from steel alloy. The composite material considered in this study is aluminum coated with various types of fibers impregnated with epoxy resin. In addition to various types of fibers, different winding directions were used in the laminate production to cover aluminum pipe as the base material. Besides these criteria, an important practical criterion, the cost criterion, is also included. Using statistical methods and based on the author's previous results, the selection of optimal parameters for the fabrication of the mentioned shaft was conducted. In conclusion, the paper draws conclusions and establishes the author's future direction of research in this field.

2. Shaft model characteristics

The data analyzed in the study pertain to the drive shaft of the TURBO ZETA 85.14B truck. The analysis was conducted for a steel shaft, an aluminum shaft (Al), and hybrid shafts made with a combination of aluminum and various composite materials (carbon fibers/epoxy resin – Al/USN, glass fibers/epoxy resin – Al/UGN, and aramid fibers/epoxy resin – Al/UKN).

The shaft has a ring-shaped cross-section with a length of 1350 mm, an outer diameter of 85 mm, and an inner diameter of 79 mm. In the case of hybrid shafts, the thickness of the aluminum tube was 2 mm, and the total thickness of eight layers of composite was 1 mm.

The analysis of the impact of the fiber orientation angles in the layers (laminae) on the analyzed parameters proved to be interesting. The parameters included the shaft torsion angle (θ), the stress due to torsion (τ), critical rotational speed (n_{kr}), and critical moment (T_{kr}). The analysed fiber orientation angles were: 0°, ±15°, ±30°, and ±45° relative to the longitudinal axis of the shaft.

The data were obtained through numerical analysis using the Finite Element Method (FEM). Modelling and analysis were conducted using FEMAP 2021.2 software. The shaft was modelled with isoperimetric quadrilateral finite elements in the form of multi-layered shells.

A visual representation of the analyzed shaft model with the generated finite element mesh can be observed in the Figure 1.

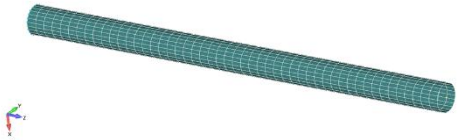

Figure 1. Shaft finite element model

The analysis was conducted for the maximum torsional moment that can occur during operation, amounting to 5000 Nm. Among the data of interest for the analysis were mass (m) and material cost. In the context of vehicles, it is well-known that reducing the vehicle's mass leads to higher speed, lower fuel consumption, and reduced CO_2 emissions, which significantly impacts environmental protection and improves the quality of life.

As one of the main factors in the production of any component, the cost of manufacturing that part becomes crucial. The cost of manufacturing typically consists of two components: the material cost and the service cost for producing the specified component. In the case of this shaft, the material cost is usually divided into three parts: the cost of the base material, the cost of fibers, and the cost of epoxy resin for binding the fibers to the base material. The cost of the laminating service is generally the same for any type of fiber. The biggest difference in cost among various composite shafts in this study is the type of fibers used for lamination.

3. Application of MCDM in the process of choosing the optimal characteristics of shaft material

Through a review of the historical development (Bana e Costa and Pirlot, 1997) of multi-criteria decision-making methods (MCDM), it can be concluded that the concept of multi-criteria decision-making was first introduced into management sciences in the United States during 1972. MCDM methods aim to reach optimal and compromise solutions based on a selected procedure, improving the quality of decisions involving the satisfaction of multiple criteria. The ultimate goal is to make the choice explicit, rational, and efficient.

Within the standard MCDM approach, a finite set of alternatives (options, potential solutions) Ai is considered. Each of the examined alternative solutions can be evaluated according to several adopted criteria (attributes) C_j. The considered alternatives (A_i), selected criteria (C_j), and their interrelations are represented in the form of a matrix table, also known as a decision matrix

Each of the selected criteria (C_j) can be of maximization (max) or minimization (min) character, depending on the type of attribute-criteria. In Table 1, the variable x_{ij} represents the value of the i-th alternative concerning the j-th criterion, and W_j is the weight coefficient of the j-th criterion. The values of alternatives according to criteria, x_{ij}, are presented numerically or through quantitative-qualitative linguistic expressions, depending on the nature of the criteria.

Table 1. Decision matrix table

		Criteria					
		$C_1(W_1)$	$C_2(W_2)$	…	$C_j(W_j)$	…	$C_n(W_n)$
Scenario alternatives	A_1	x_{11}	x_{12}	…	x_{1j}	…	x_{1n}
	A_2	x_{21}	x_{22}	…	x_{2j}	…	x_{2n}
	⋮	⋮	⋮	…	⋮	…	⋮
	A_i	x_{i1}	x_{i2}	…	x_{ij}	…	x_{in}
	⋮	⋮	⋮	…	⋮	…	⋮
	A_m	x_{m1}	x_{m2}	…	x_{mj}	…	x_{mn}
	max/min	max	max	…	min	…	min

The selection of weight coefficients (W_j) is a subjective process and depends on the views and preferences of decision-makers. These coefficients determine the importance of each criterion in the process of selection and ranking. There is absolute and relative importance of weight coefficients, and their selection can be a challenging aspect of the MCDM methodology.

As a conclusion of the MCDM process, the stability of the solution concerning changes in input data and weight coefficients is usually examined. If the order of alternatives remains stable, the results of the MCDM process are considered reliable. Sensitivity analysis is an essential aspect that can serve for additional comparison of very similar alternatives and resolving conflicts between decision-makers.

In summary, the MCDM process enables quantitative and qualitative ranking of different materials. In this paper, the SAW and TOPSIS methods were applied to select optimal alternatives for a specific application.

3.1. Choosing the optimal characteristics of shaft material using a SAW method

SAW (Simple Additive Weighting Method) - The Simple Additive Weighting Method (Fishburn, 1967) is a straightforward and commonly applied method that yields similar results to much more complex multi-criteria decision-making methods. This method belongs to the category of Multiple Attribute Decision Making (MADM) methods, specifically the subgroup of Utility methods (Chai et al., 2013; Figueira et al., 2005; Hwang et al., 1981; Polatidis et al., 2006; Yoon, 1980). This approach takes into account the weight coefficients of criteria. Each criterion needs to be associated with a weight factor (ponder) assigned directly by the decision-maker or obtained using one of the well-known methods for determining the weight coefficients of criteria. For each considered alternative, the Comprehensive Characteristic is calculated, representing the sum of products of relative weight factors and normalized performance values across all criteria. The alternative with the highest value represents the best solution among those offered:

$$A^* = \left\{ A_i \,\middle|\, \max_i \sum_{j=1}^{n} W_j' r_{ij} \right\}, \qquad (1)$$

where: Wj' represents normalized value of the weight coefficient W_j:

$$W_j' = \frac{W_j}{\sum_{j=1}^{n} W_j}. \qquad (2)$$

The r_{ij} values are obtained through a linear normalization procedure.

In the Table 2 for each of the 14 observed materials, the values for 6 selected characteristics are presented (θ – twist angle, twisting stress, shat critical RPM, shaft critical torque, shaft mass and shaft manufacturing cost).

Table 2.Values of chosen characteristics for the proposed shaft materials

Material	θ, rad	τ, MPa	n_{kr}, min-1	T_{kr}, Nm	m, kg	Price, €
Steel (A1)	0.068	168	8472	32645	7.87	13.57
Al (A2)	0.202	166.7	8496	11090	2.61	32.47
Al/[$0_{USN.8}$] (A3)	0.222	150.9	9996	18631	2.27	59.48
Al/[$\pm 15_{USN.4}$] (A4)	0.204	140.2	9810	17821	2.27	60.81
Al/[$\pm 30_{USN.4}$] (A5)	0.174	122.5	9270	15249	2.27	62.29
Al/[$\pm 45_{USN.4}$] (A6)	0.161	114.4	8616	11609	2.27	63.94
Al/[$0_{UGN.8}$] (A7)	0.224	151.6	8328	12870	3.07	47.87
Al/[$\pm 15_{UGN.4}$] (A8)	0.218	148.3	8268	12395	3.07	47.93
Al/[$\pm 30_{UGN.4}$] (A9)	0.207	142.1	8112	11279	3.07	47.98
Al/[$\pm 45_{UGN.4}$] (A10)	0.202	139	7944	10122	3.07	48.04
Al/[$0_{UKN.8}$] (A11)	0.23	155.6	9306	15627	2.02	68.48
Al/[$\pm 15_{UKN.4}$] (A12)	0.217	147.8	9174	14886	2.02	70.82
Al/[$\pm 30_{UKN.4}$] (A13)	0.193	134.2	8802	13221	2.02	73.41
Al/[$\pm 45_{UKN.4}$] (A14)	0.182	127.4	8364	12133	2.02	76.3
Criteria type	min	min	max	max	min	min

For the normalization of the data from the Table 2, in dependence of the criteria type, the following equations are used:

$$r_{ij} = \frac{x_{ij}-x_j^{min}}{x_j^{max}-x_j^{min}}, \qquad (3)$$

$$r_{ij} = \frac{x_j^{max}-x_{ij}}{x_j^{max}-x_j^{min}}. \qquad (4)$$

where: equation (3) is used for criteria of maximum type, and equation (4) is used for the criteria of minimum type.

Normalized data is given in Table 3.

Table 3. Normalized data from

	C1	C2	C3	C4	C5	C6
A1	1	0	0.25731	1	0	1
A2	0.17284	0.024254	0.269006	0.042978	0.899145	0.698709
A3	0.049383	0.31903	1	0.377792	0.957265	0.268133
A4	0.160494	0.518657	0.909357	0.341828	0.957265	0.246931
A5	0.345679	0.848881	0.646199	0.227634	0.957265	0.223338
A6	0.425926	1	0.327485	0.066021	0.957265	0.197035
A7	0.037037	0.30597	0.187135	0.122009	0.820513	0.453212
A8	0.074074	0.367537	0.157895	0.100919	0.820513	0.452256
A9	0.141975	0.483209	0.081871	0.05137	0.820513	0.451459
A10	0.17284	0.541045	0	0	0.820513	0.450502
A11	0	0.231343	0.663743	0.244417	1	0.124661
A12	0.080247	0.376866	0.599415	0.211517	1	0.087359
A13	0.228395	0.630597	0.418129	0.137593	1	0.04607
A14	0.296296	0.757463	0.204678	0.089287	1	0

In the process of determining the weight coefficients of selected criteria, the well-known Saaty scale was applied, enabling the comparison of criteria in pairs by translating linguistic expressions into numbers. Saaty scale is given in Table 4.

Table 4.Criteria comparison in pairs – Saaty scale

Linguistics expression	Qualitatively-number value of ratio $W_{k`}/ W_{k``}$
Equal importance of criteria *K*` and *K*``	1
Slightly bigger importance of *K*` related to *K*``	3
Much bigger importance of *K*` related to *K*``	5
Dominant importance of *K*` related to *K*``	7
Absolute importance of *K*` related to *K*``	9
Intermediate statements – statements between two adjacent statements	2, 4, 6 and 8

In accordance with the Saaty procedure, four different Dominance matrices were formed, with varying degrees of importance relationships between the selected criteria (from Table 5 to Table 8). In the Dominance matrices, data were entered based on the values from Table 4 and if criterion K' is less important than criterion K'', then a value of zero is entered in that field.

In the first three Dominance matrices, the criteria had different importance, and consequently, different weight coefficients. In the last, fourth case (Table 8), all criteria had equal importance, and therefore, the weight coefficients had identical values.

Table 5. Dominance matrix 1

	k1	k2	k3	k4	k5	k6	W	W'
k1	1	2	0	5	3	3	14	0.25
k2	0	1	0	3	2	2	8	0.142857
k3	2	4	1	7	5	5	24	0.428571
k4	0	0	0	1	0	0	1	0.017857
k5	0	0	0	2	1	0	3	0.053571
k6	0	0	0	3	2	1	6	0.107143
Σ							56	1

Table 6. Dominance matrix 2

	k1	k2	k3	k4	k5	k6	W	W'
k1	1	3	0	3	0	0	7	0.104478
k2	0	1	0	1	0	0	2	0.029851
k3	3	5	1	5	1	0	15	0.223881
k4	0	1	0	1	0	0	2	0.029851
k5	3	5	1	5	1	0	15	0.223881
k6	5	7	3	7	3	1	26	0.38806
Σ							67	1

Table 7. Dominance matrix 3

	k1	k2	k3	k4	k5	k6	W	W'
k1	1	1	0	1	0	3	6	0.1
k2	1	1	0	1	0	3	6	0.1
k3	3	3	1	3	0	5	15	0.25
k4	1	1	0	1	0	3	6	0.1
k5	5	5	3	5	1	7	26	0.433333
k6	0	0	0	0	0	1	1	0.016667
Σ							60	1

Table 8. Dominance matrix 4

	k1	k2	k3	k4	k5	k6	W	W'
k1	1	1	1	1	1	1	6	0.166667
k2	1	1	1	1	1	1	6	0.166667
k3	1	1	1	1	1	1	6	0.166667
k4	1	1	1	1	1	1	6	0.166667
k5	1	1	1	1	1	1	6	0.166667
k6	1	1	1	1	1	1	6	0.166667
Σ							36	1

Subsequently, based on the previously mentioned values, aggregate characteristics were determined (using the equation (1)) for all 14 examined materials, for each of the four defined variants of weight coefficient distributions. Diagrams with the values of the obtained Aggregate Characteristics are shown in images (from Figure 2 to Figure 5).

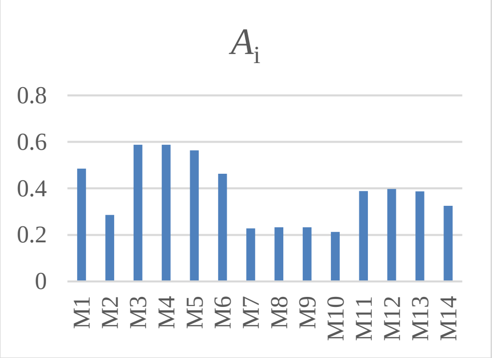

Figure 2. Values of Summary characteristics for the first variant of the weighting coefficients (Table 5)

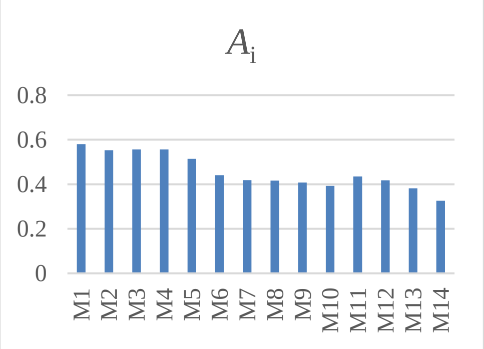

Figure 3. Values of Summary characteristics for the second variant of the weighting coefficients (Table 6)

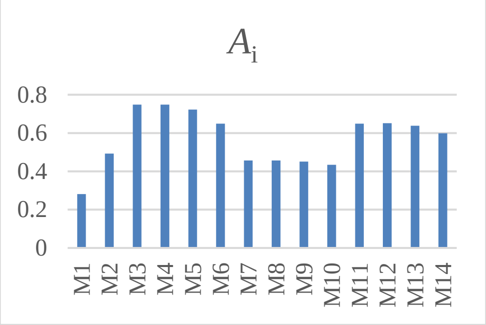

Figure 4. Values of Summary characteristics for the third variant of the weighting coefficients (Table 7)

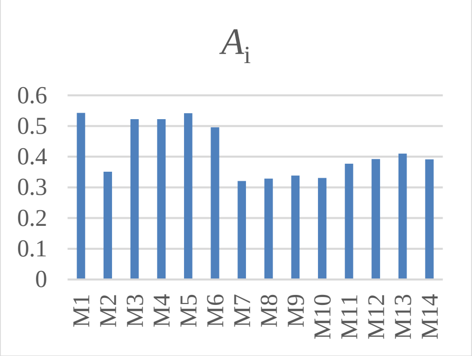

Figure 5. Values of Summary characteristics for the fourth variant of the weighting coefficients (Table 8)

3.2. Choosing the optimal characteristics of shaft material using a TOPSIS method

In the second part of the analysis – the selection of the optimal material, the TOPSIS (Technique for Order Preference by Similarity to Ideal Solution) method was used. This method evaluates the considered alternatives (materials) based on their distances (Euclidean distance) from the so-called "ideal" and "anti-ideal" solutions (Hwang et al., 1981). The best alternative is chosen as the one that, in a combined sense, has the smallest Euclidean distance from the "ideal" and the largest from the "anti-ideal" solution.

In the first step, normalization of the values x_{ij} from the initial matrix (Table 2) is performed based on the following equation:

$$r_{ij} = \frac{x_{ij}}{\sqrt{\sum_{i=1}^{6} x_{ij}^{2}}}. \qquad (5)$$

In Table 9 are shown normalized values of r_{ij} which made normalized matrix R, and common for all weight coefficient values.

Table 9. Normalized values from Table 2, according to equation (5) – *R* matrix

	C1	C2	C3	C4	C5	C6
A1	0.096281	0.346332	0.276158	0.670745	0.910982	0.063744
A2	0.286011	0.343652	0.27694	0.227862	0.302117	0.152526
A3	0.314329	0.311081	0.325835	0.382804	0.262761	0.279404
A4	0.288843	0.289022	0.319772	0.366162	0.262761	0.285652
A5	0.246366	0.252534	0.30217	0.313316	0.262761	0.292604
A6	0.227959	0.235836	0.280852	0.238526	0.262761	0.300355
A7	0.317161	0.312524	0.271464	0.264435	0.355364	0.224867
A8	0.308665	0.305721	0.269508	0.254675	0.355364	0.225149
A9	0.293091	0.292939	0.264423	0.231745	0.355364	0.225384
A10	0.286011	0.286549	0.258947	0.207973	0.355364	0.225665
A11	0.325656	0.32077	0.303343	0.321082	0.233823	0.321681
A12	0.30725	0.30469	0.29904	0.305857	0.233823	0.332673
A13	0.273268	0.276653	0.286915	0.271647	0.233823	0.34484
A14	0.257693	0.262635	0.272637	0.249292	0.233823	0.358415

Elements of the named difficult normalized matrix *V* - v_{ij}, whose values are shown in the Table 10 to Table 13, are obtained based on the following equations:

$$W_j^{`} = \frac{W_j}{\sum_{j=1}^{6} W_j} \; and \; v_{ij} = W_j^{`} \cdot r_{ij} \qquad (6)$$

Table 10. Weighted normalized matrix for the first variant of the weighting coefficients

	C1	C2	C3	C4	C5	C6
A1	0.024070243	0.049476	0.118353	0.011978	0.048803	0.00683
A2	0.071502781	0.049093	0.118689	0.004069	0.016185	0.016342
A3	0.078582264	0.04444	0.139643	0.006836	0.014076	0.029936
A4	0.072210729	0.041289	0.137045	0.006539	0.014076	0.030606
A5	0.061591504	0.036076	0.129501	0.005595	0.014076	0.03135
A6	0.05698984	0.033691	0.120365	0.004259	0.014076	0.032181
A7	0.079290212	0.044646	0.116342	0.004722	0.019037	0.024093
A8	0.077166367	0.043674	0.115503	0.004548	0.019037	0.024123
A9	0.073272652	0.041848	0.113324	0.004138	0.019037	0.024148
A10	0.071502781	0.040936	0.110977	0.003714	0.019037	0.024178
A11	0.081414057	0.045824	0.130004	0.005734	0.012526	0.034466
A12	0.076812393	0.043527	0.12816	0.005462	0.012526	0.035644
A13	0.068317013	0.039522	0.122963	0.004851	0.012526	0.036947
A14	0.064423297	0.037519	0.116845	0.004452	0.012526	0.038402
Crit. type	min	min	max	max	min	min

Table 11. Weighted normalized matrix for the second variant of the weighting coefficients

	C1	C2	C3	C4	C5	C6
A1	0.010059	0.010338	0.061826	0.020022	0.203951	0.024737
A2	0.029882	0.010258	0.062001	0.006802	0.067638	0.059189
A3	0.03284	0.009286	0.072948	0.011427	0.058827	0.108426
A4	0.030178	0.008628	0.071591	0.01093	0.058827	0.11085
A5	0.02574	0.007538	0.06765	0.009353	0.058827	0.113548
A6	0.023817	0.00704	0.062877	0.00712	0.058827	0.116556
A7	0.033136	0.009329	0.060775	0.007894	0.079559	0.087262
A8	0.032249	0.009126	0.060338	0.007602	0.079559	0.087371
A9	0.030621	0.008744	0.059199	0.006918	0.079559	0.087462
A10	0.029882	0.008554	0.057973	0.006208	0.079559	0.087572
A11	0.034024	0.009575	0.067913	0.009585	0.052348	0.124832
A12	0.032101	0.009095	0.066949	0.00913	0.052348	0.129097
A13	0.02855	0.008258	0.064235	0.008109	0.052348	0.133818
A14	0.026923	0.00784	0.061038	0.007442	0.052348	0.139087
Crit. type	min	min	max	max	min	min

Table 12. Weighted normalized matrix for the third variant of the weighting coefficients

	C1	C2	C3	C4	C5	C6
A1	0.009628	0.034633	0.069039	0.067074	0.394759	0.001062
A2	0.028601	0.034365	0.069235	0.022786	0.130918	0.002542
A3	0.031433	0.031108	0.081459	0.03828	0.113863	0.004657
A4	0.028884	0.028902	0.079943	0.036616	0.113863	0.004761
A5	0.024637	0.025253	0.075542	0.031332	0.113863	0.004877
A6	0.022796	0.023584	0.070213	0.023853	0.113863	0.005006
A7	0.031716	0.031252	0.067866	0.026444	0.153991	0.003748
A8	0.030867	0.030572	0.067377	0.025468	0.153991	0.003752
A9	0.029309	0.029294	0.066106	0.023175	0.153991	0.003756
A10	0.028601	0.028655	0.064737	0.020797	0.153991	0.003761
A11	0.032566	0.032077	0.075836	0.032108	0.101323	0.005361
A12	0.030725	0.030469	0.07476	0.030586	0.101323	0.005545
A13	0.027327	0.027665	0.071729	0.027165	0.101323	0.005747
A14	0.025769	0.026264	0.068159	0.024929	0.101323	0.005974
Crit. type.	min	min	max	max	min	min

Table 13. Weighted normalized matrix for the fourth variant of the weighting coefficients

	C1	C2	C3	C4	C5	C6
A1	0.016047	0.057722	0.046026	0.111791	0.15183	0.010624
A2	0.047669	0.057275	0.046157	0.037977	0.050353	0.025421
A3	0.052388	0.051847	0.054306	0.063801	0.043794	0.046567
A4	0.04814	0.04817	0.053295	0.061027	0.043794	0.047609
A5	0.041061	0.042089	0.050362	0.052219	0.043794	0.048767
A6	0.037993	0.039306	0.046809	0.039754	0.043794	0.050059
A7	0.05286	0.052087	0.045244	0.044073	0.059227	0.037478
A8	0.051444	0.050953	0.044918	0.042446	0.059227	0.037525
A9	0.048848	0.048823	0.04407	0.038624	0.059227	0.037564
A10	0.047669	0.047758	0.043158	0.034662	0.059227	0.037611
A11	0.054276	0.053462	0.050557	0.053514	0.03897	0.053614
A12	0.051208	0.050782	0.04984	0.050976	0.03897	0.055446
A13	0.045545	0.046109	0.047819	0.045275	0.03897	0.057473
A14	0.042949	0.043773	0.04544	0.041549	0.03897	0.059736
Crit. type.	min	min	max	max	min	min

In the third step of the multi-criteria analysis using the TOPSIS method, the formation of the so-called "ideal" and "anti-ideal" solutions is approached. The ideal solution (A^+) (Table 14) possesses all the best characteristics across all criteria (parameters), all of which belong to the *min* type. It is determined based on the equation:

$$A^+ = \left\{\left(\max_i v_{ij} \,\middle|\, j \in C'\right) \cup \left(\min_i v_{ij} \,\middle|\, j \in C''\right)\right\} = \left\{v_1^+, v_2^+, \dots, v_j^+, \dots, v_6^+\right\}, i = \overline{1.6} \qquad (7)$$

On the other hand, the anti-ideal solution (A^-) (Table 14) consists of all the worst characteristics across all criteria (parameters) and is determined according to the equation:

$$A^- = \left\{\left(\min_i v_{ij} \,\middle|\, j \in C'\right) \cup \left(\max_i v_{ij} \,\middle|\, j \in C''\right)\right\} = \left\{v_1^-, v_2^-, \dots, v_j^-, \dots, v_6^-\right\}, i = \overline{1.6} \qquad (8)$$

Of the selected criteria, four are of the min type (C_1, C_2, C_5, and C_6), while two are of the max type (C_3 and C_4).

Table 14. Ideal and anti-ideal solution for all four variants

Variant	Ideal/anti-ideal solution	C1	C2	C3	C4	C5	C6
1	A^+	0.024070243	0.033691	0.139643	0.011978	0.012526	0.00683
	A^-	0.081414057	0.049476	0.110977	0.003714	0.048803	0.038402
2	A^+	0.010059	0.00704	0.072948	0.020022	0.052348	0.024737
	A^-	0.034024	0.010338	0.057973	0.006208	0.203951	0.139087
3	A^+	0.009628	0.023584	0.081459	0.067074	0.101323	0.001062
	A^-	0.032566	0.034633	0.064737	0.020797	0.394759	0.005974
4	A^+	0.016047	0.039306	0.054306	0.111791	0.03897	0.010624
	A^-	0.054276	0.057722	0.043158	0.034662	0.15183	0.059736

The fourth step of the TOPSIS analysis involves determining the distance (Euclidean distance) (Table 15) of each alternative (A_i) from the ideal and anti-ideal solutions. The distance from the ideal solution is obtained using the equation:

$$D_i^+ = \sqrt{\sum_{j=1}^{6}\left(v_{ij} - v_j^+\right)^2} \;, \qquad (9)$$

and from anti-ideal:

$$D_i^- = \sqrt{\sum_{j=1}^{6}\left(v_{ij} - v_j^-\right)^2} \;. \qquad (10)$$

Table 15. Euclidian distance from ideal and anti-ideal solution for all four variants

Alternative (material)	Weight coefficient variant							
	1		2		3		4	
	D_1^+	D_1^-	D_2^+	D_2^-	D_3^+	D_3^-	D_4^+	D_4^-
A1	0.044927	0.066391	0.152046	0.117711	0.293906	0.052061	0.114652	0.099149
A2	0.055611	0.041334	0.046029	0.158109	0.058865	0.263939	0.084772	0.10742
A3	0.060414	0.046287	0.087428	0.149181	0.039132	0.281963	0.071388	0.113388
A4	0.054583	0.04589	0.089159	0.148606	0.03873	0.281836	0.071258	0.112897
A5	0.046484	0.046635	0.0912	0.147964	0.041381	0.281571	0.075339	0.112122
A6	0.046476	0.046677	0.094498	0.147338	0.048382	0.281354	0.085472	0.111396
A7	0.064073	0.033887	0.074047	0.134802	0.071857	0.24089	0.085545	0.095903
A8	0.062436	0.034099	0.073983	0.134759	0.072184	0.240878	0.086136	0.095845
A9	0.059866	0.034918	0.073901	0.134739	0.07324	0.240875	0.088095	0.095875
A10	0.059462	0.035489	0.074155	0.134713	0.074754	0.240885	0.091012	0.095957
A11	0.065812	0.041363	0.103605	0.152635	0.043255	0.293875	0.083188	0.114905
A12	0.062314	0.040969	0.107405	0.152242	0.043461	0.293806	0.084234	0.114561
A13	0.056814	0.041637	0.111625	0.151948	0.045159	0.293718	0.08705	0.114403
A14	0.056712	0.042221	0.116881	0.151826	0.047381	0.293683	0.090378	0.114512

In the penultimate, fifth step of the analysis, the calculation of the relative closeness of each alternative to the ideal solution is performed:

$$RC_i = \frac{D_i^-}{D_i^- + D_i^+}. \qquad (11)$$

The values of RC_i for each of the fourteen considered materials are shown in Table 16, for all four variants of selected weighting coefficients. Additionally, the same table displays the rank of the corresponding alternative based on 4 different variants of weighting coefficients.

Table 16. The relative closeness of all fourteen considered alternatives (materials) for the four variants is as follows

Alternative (material)	Weight coefficient variant							
	1		2		3		4	
	RC_i	Rank	RC_i	Rank	RC_i	Rank	RC_i	Rank
A1	0.59641	1	0.436359	14	0.150479	14	0.463744	14
A2	0.426366	7	0.774521	1	0.817645	9	0.558921	8
A3	0.433799	5	0.630496	6	0.878129	2	0.613652	1
A4	0.456743	4	0.625013	7	0.879182	1	0.613055	2
A5	0.500807	3	0.618671	8	0.871866	3	0.59811	3
A6	0.501079	2	0.609248	9	0.853271	8	0.565841	7
A7	0.34593	14	0.645454	4	0.770239	10	0.528543	10
A8	0.353228	13	0.645577	3	0.769427	11	0.526673	11
A9	0.368398	12	0.645796	2	0.766838	12	0.521146	12
A10	0.373758	11	0.644967	5	0.763167	13	0.513226	13
A11	0.385941	10	0.595672	10	0.871696	4	0.580056	4
A12	0.396667	9	0.586342	11	0.871137	5	0.576277	5
A13	0.422923	8	0.576494	12	0.86674	6	0.567887	6
A14	0.426763	6	0.565025	13	0.861079	7	0.558894	9

On the diagrams shown in Figure 6 to Figure 9, the values of the relative closeness of alternatives given in Table 16 are displayed.

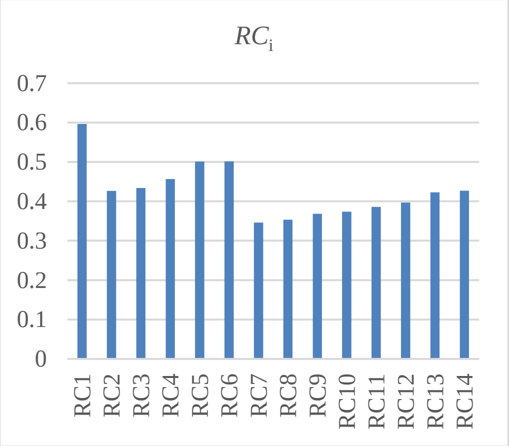

Figure 6. Relative Closeness Values (RC_i) for the first variant of the weighting coefficients (Table 16)

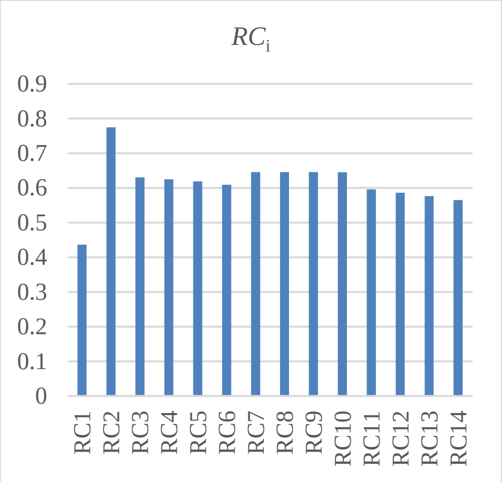

Figure 7. Relative Closeness Values (RC_i) for the second variant of the weighting coefficients (Table 16)

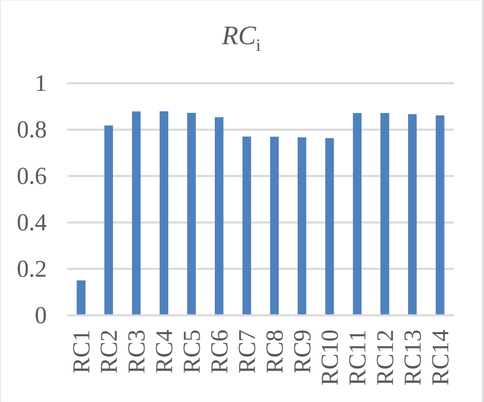

Figure 8. Relative Closeness Values (*RC*$_i$) for the third variant of the weighting coefficients (Table 16)

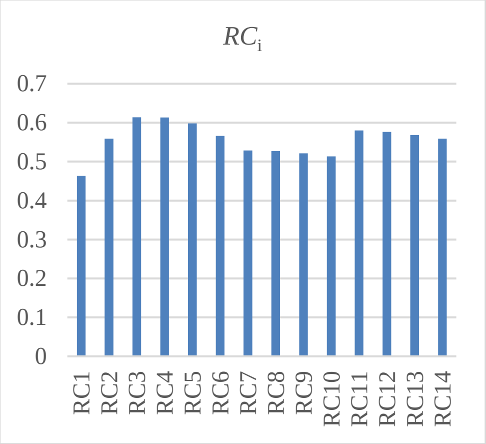

Figure 9. Relative Closeness Values (*RC*$_i$) for the fourth variant of the weighting coefficients (Table 16)

At the end of the analysis and selection of the optimal material, the calculation of the average values of Aggregate Characteristics (*A*$_i$) was performed for all four variants of weight coefficients (SAW method), as well as the average values of Relative Closeness (*RC*$_i$) for all four variants using the TOPSIS method. These average values, in a way, facilitate the decision-maker - the constructor, to perceive, in terms of the selected different weight coefficients of characteristics-criteria, a general evaluation of alternative materials and their ranking.

The diagram shown in the Figure 10 represents the average values of Aggregate Characteristics (*A*$_i$) for all four variants of the analysis done with the SAW method. Average values of Relative closeness - *RC*$_i$ (TOPSIS method) for the fourteen considered materials is shown in the Figure 11.

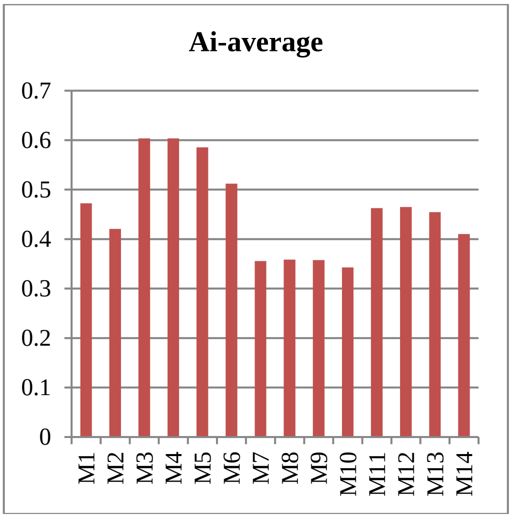

Figure 10. Average values of Aggregate characteristics - *A*$_i$ (SAW method) for the fourteen considered materials

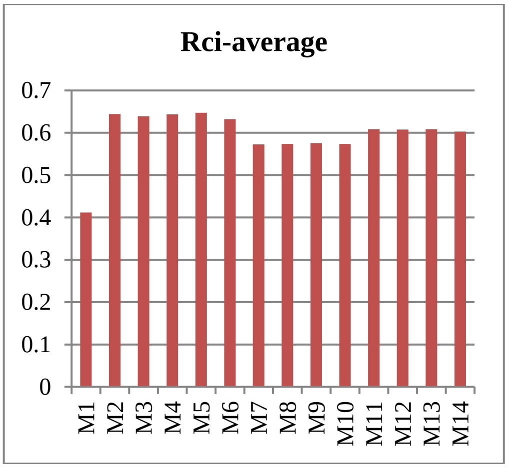

Figure 11. Average values of Relative closeness - *RC*$_i$ (TOPSIS method) for the fourteen considered materials

4. Results discussion

The process of selecting the optimal material was implemented using two different multi-criteria decision-making (MCDM) methods – Simple Additive Weighting (SAW) and Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS). The SAW method demonstrated significant superiority for hybrid shafts made of aluminum and composite material – carbon fibers/epoxy resin, especially for fiber orientation of 0^0 and $\pm 15^0$ relative to the longitudinal axis of the shaft. Additionally, hybrid shafts made of aluminum and composite material with aramid fibers/epoxy resin (for fiber orientation angles of 0^0, $\pm 15^0$, and $\pm 30^0$) showed relatively good values for the Comprehensive Characteristic (A_i). On the other hand, steel proved to be a good choice in two variants of weight coefficients (second and fourth), and in one, it showed solid performance (first variant). However, the significantly lower value of A_i in the third variant was influenced by the notably higher mass of steel compared to other analyzed materials, with this characteristic being given high importance through a high weight coefficient.

According to the SAW analysis, the hybrid shaft group potentially made of glass fibers/epoxy resin exhibited the least favorable characteristics. The shafts in this group showed small differences in the values of the Comprehensive Characteristic, indicating relatively minor effects of fiber orientation. Nevertheless, the variant with fiber orientation at $\pm 45^0$ proved to be potentially the least favorable. Regarding aluminum shafts, they were well-evaluated in the case where special importance was given to mass and material cost criteria – the variant with 2 weight coefficients. In other cases, aluminum shafts would not be a sufficiently good choice.

In the second part of the analysis, through the application of the TOPSIS method, the results were largely consistent with those obtained using the SAW method regarding the ranking of the considered materials. However, values of the Relative Closeness characteristic were more uniform for all materials in the TOPSIS analysis, with fewer significant differences. This is likely a consequence of the philosophy of the TOPSIS method, which analyzes the deviation of each alternative from both the "ideal" and "anti-ideal" solutions. In this case, the group of hybrid shafts made of aluminum and composite material – carbon fibers/epoxy resin showed the best results for all fiber orientation angles, especially for $\pm 15^0$ and $\pm 30^0$. The TOPSIS analysis provided consistent and good values for the Relative Closeness for the group of hybrid shafts made of aramid fibers/epoxy resin. Similar to the SAW procedure, the least favorable solution was identified as the shafts made of glass fibers/epoxy resin, with noticeable differences in the obtained ranking results.

A noticeable difference between the obtained ranking results for both methods is related to steel and aluminum as potential materials. The TOPSIS method gives preference to aluminum as a material (second-ranked material out of 14 analyzed), while SAW favors steel. The concept of the TOPSIS method, through selected weight coefficients, results in a better rating for aluminum, which does not have an explicitly poor characteristic like steel (criteria C1 and C5). Consequently, in some variants of weight coefficients, steel is much closer to the "anti-ideal" solution. Due to its solid performance, with more good than bad characteristics but without extremes, aluminum as a potential choice for shaft material takes a high second place in the TOPSIS ranking.

In conclusion, summarizing the results of both conducted analyses, materials M3, M4, and M5 could be the final optimal-compromised choice for the fabrication of the mentioned shaft. On the other hand, designers should avoid the application of materials labeled M7-M10 in this particular case.

5. Conclusions

Ranking and selecting the optimal material are crucial aspects of the product design process. The selection process cannot rely solely on the free judgment of the designer; instead, it should involve a specific decision-making procedure, implemented using one of the multi-criteria decision-making (MCDM) methods. In the analysis and material selection process from a larger pool of considerations, it is necessary to define relevant criteria, to which specific weight coefficients are assigned during the analysis, in line with the requirements within the product design and usage process. In this study, in addition to mechanical and physical characteristics, the cost of manufacturing the mentioned shaft was included as a criterion. Two MCDM methods, Simple Additive Weighting (SAW) and Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS), were used for ranking and material selection. A total of 14 potential shaft materials (including steel, aluminum, and 12 hybrid variants with composite applications and different fiber orientations) were analyzed and evaluated through the selection of six chosen criteria.

To conduct a sensitivity analysis of the results, four groups of weight coefficients were formed. Both methods showed fairly good agreement in terms of material ranking and the selection of the optimal variant. The overall review of the values of Comprehensive Characteristics (A_i, SAW method) and Relative Closeness (R_{ci}, TOPSIS method) indicates that the group of hybrid shafts made of aluminum and composite material – carbon fibers/epoxy resin (M3, M4, and M5) was the best-rated option. The application of two methods, whose results largely coincide, provides additional confidence to the designer when selecting the appropriate material.

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